Lesson 1

How Does the Surface Area of a Rectangular Prism Change with its Linear Dimensions?

Grade Level: 5-8

BACKGROUND

Scientists at NASA are building a robot called the Personal Satellite Assistant, or PSA. The PSA's mission is to keep the astronauts safe and to assist them with their chores on space-based vehicles such as the International Space Station (ISS), a Crew Exploration Vehicle, or even on Mars. This small round robot will float in microgravity and move autonomously (without direction from people). It will keep track of the astronauts' schedules, monitor supplies, assist with scientific experiments, communicate with Mission Control, and help keep the astronauts safe by monitoring the air composition and temperature.

NASA engineers have created a model of the PSA with a diameter of 12 inches (30.5 cm). The engineers' goal is to build a PSA with an 8-inch (20-cm) diameter, because it will be safer and will require less power to move around.

INTRODUCTION

In this lesson, we assume that the computer inside the PSA is in the shape of a rectangular prism. The prism must be made smaller in order to fit into a PSA with an 8-inch (20-cm) diameter. Students discover that the surface area of a rectangular prism changes as its length, width, and height change. They find that the surface area increases as you flatten the prism, and that the width of the 8-inch (20-cm) sphere in which it must fit restricts the length, width, and height of the prism. They also discover that there is more than one solution to this problem.

MAIN CONCEPT

The surface area of a rectangular prism of fixed volume increases as you flatten it.

NASA RELEVANCE

NASA scientists and engineers working on the PSA project need to reduce the volume and mass of the PSA because of the high cost of transportation for leaving our home planet, limited space in space-based vehicles, and for safety reasons. Even though they use computer programs to design the PSA and its components, a basic understanding of volume and surface area is essential in order to design the PSA and the shape of the components that go inside.

PREREQUISITE SKILLS

Students should be able to:

- Calculate the surface area and volume of a rectangular prism.
- Add and multiply decimals.
- Recognize a rectangular prism and measure its dimensions.
- Measure the diameter of a sphere.

INSTRUCTIONAL OBJECTIVES

During this lesson, students will:

1. Find the maximum surface area of a rectangular prism that has a volume of 24 cubic inches (393 cm^3) and fits into a sphere with a diameter of 8 inches (20 cm).
2. Write a letter to NASA engineers with the recommended dimensions for a computer for a PSA with an explanation of why this is the best solution.
3. Explain their recommendation of how to stack extra PSA computer components.

NATIONAL EDUCATION STANDARDS		
Fully Met	*Partially Met*	*Addressed*
NCTM Mathematics Standards *Geometry (6-8)* Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. *Measurement (6-8)* Apply appropriate techniques, tools, and formulas to determine measurements. **Project 2061** *Benchmark 9B (6-8): The Mathematical World (Symbolic Relationships) #2* Mathematical statements can be used to describe how one quantity changes when another changes.	**NCTM Mathematics Standards** *Geometry (6-8)* Use visualization, spatial reasoning, and geometric modeling to solve problems. **Project 2061** *Benchmark 3B (6-8): The Nature of Technology (Design and Systems) #1* Design usually requires taking constraints into account.	**NCTM Mathematics Standards** *Measurement (6-8)* Understand measurable attributes of objects and the units, systems, and processes of measurement **Project 2061** *Benchmark 2B (6-8): The Nature of Mathematics (Mathematics, Science, and Technology) #1* Mathematics is helpful in almost every human endeavor – from laying bricks to prescribing medicine or drawing a face. In particular, Mathematics has contributed to progress in science and technology for thousands of years and still continues to do so. *Benchmark 9C (6-8): The Mathematical World (Shapes) #1* Some shapes have special properties: Triangular shapes tend to make structures rigid, and round shapes give the least possible boundary for a given amount of interior area. **ITEA Standards for Technological Literacy** Standard 8: Students will develop an understanding of the attributes of design. **National Science Education Standards** *E (5-8) Science and Technology #1* Abilities of technological design.

MAJOR CONCEPTS

- In technology design, constraints often must be taken into consideration.
- Some shapes have special properties.
- Mathematical statements can be used to describe how one quantity changes when another changes.
- There are many possible solutions to a problem.

PREPARING FOR THE ACTIVITY

- Have the students view the video clips on the PSA Web site before starting the lesson. These can also be viewed whole class if there is access to a computer projector. (See http://psa.arc.nasa.gov/sles.shtmlfor video clips.
- You might also consider having students watch the NASA CONNECT™ program “PSA: The Astronauts’ Helper.” (See http://ali.apple.com/ali_sites/ali/exhibits/1001204/)

Materials

- 24 wooden or plastic blocks with a volume of 1 cubic inch (about 16 cubic centimeters) per group of 3 or 4 students. Alternatively, this can be done as a whole-class activity using an overhead projector with an 8-inch diameter circle drawn on the transparency. (Students can arrange the cubes on the projector to find solutions.)
- One set of card stock cut into circles with an 8-inch (20-cm) diameter
- Transparent sphere (like a hamster ball) to aid in visualization (optional)

- Rulers or tape measures for each group
- Sheets of drawing paper and pencils
- Graph paper
- Calculators (optional)
- 1 copy of the Student Handout sheet for each student or group of students
- Computers with Internet* access
- Computer with Internet* access connected to projector (optional)

*** System Requirements to Run PSA Web Site Activities**

Platform	Browser
Windows 95 Windows 98 Windows Me	Internet Explorer 4.0 or later (Internet Explorer 5.0 or later is recommended), Netscape Navigator 4 or later, Netscape 7.0 or later (Netscape 6 is not recommended) JavaScript enabled
Windows NT Windows 2000	Internet Explorer 4.0 or later, Netscape Navigator 4 or later, Netscape 7.0 or later, with standard install defaults (Netscape

Windows XP or later	6 is not recommended) JavaScript enabled
Macintosh: System 8.6 System 9.0 System 9.1 System 9.2	Netscape 4.5 or later (Netscape Communicator 4.7 or Netscape 7.0 are recommended), Netscape 7.0 or later, (Netscape 6 is not recommended) Microsoft Internet Explorer 5.0 or later JavaScript enabled
Macintosh OS X 10.1 or later	Netscape 7.0 or later (Netscape 6 is not recommended), Microsoft Internet Explorer 5.1 or later JavaScript enabled
Browser plug-ins	Flash Player 6 or higher QuickTime Player 6 or higher

Time for Activity

- 1-2 class periods

An additional 30 minutes will be required to watch and discuss the video clips, and 40 minutes to watch and discuss the NASA CONNECT™ program.

Lesson Description

ENGAGE

Before this lesson, students should view the video clips on the Student page of the Personal Satellite Assistant (PSA) Web site at http://psa.arc.nasa.gov/ by clicking the "Students" tab to go to http://psa.arc.nasa.gov/stud.shtml. From this page, have students click the link to the "Using Diameter, Volume and Surface Area to Determine Dimensions of PSA's Computer." On this page, have students review the "Introductory video" that describes the PSA, the "Main Problem video," and the "Prerequisites video." They can also review the text that summarizes the problem, prerequisites, materials, and procedure.

Note: Located in the Teacher Section of the Web site, there is a teacher version of this page with additional video clips of students doing this activity and their conclusions. From the main page, click the Teacher tab to go to http://psa.arc.nasa.gov/teac.shtml. Then click the "Middle School" button. The link to the page with the teacher video clips is called "video support" and is located next to the link to "Lesson 1."

You might also consider having students view the program, NASACONNECT™: PSA: The Astronauts' Helper, available at http://ali.apple.com/ali_sites/ali/exhibits/1001204/. Working in groups, students can answer and discuss all inquiry-based questions that are presented in this half-hour program. Discuss the uses of the PSA and how it will be helpful for astronauts.

Ask students why NASA scientists need to make a smaller PSA and a smaller computer inside it. Ask them to explain what surface area is and why the surface area of the computer in the PSA needs to be as large as possible. Point out that engineers often have to take constraints into consideration when designing new technologies.

Ask students to identify the constraints or limitations they must consider in solving this problem:

1. The total volume of the rectangular prism must be 24 cubic inches (about 393 cubic centimeters).
2. The rectangular prism must fit into a sphere with a diameter of 8 inches (about 20 centimeters). This means that the linear dimensions (length, width, and height) must be less than 8 inches (20 cm).
3. The rectangular prism must have as much surface area as possible.

Review with students how to calculate the volume and surface area of a rectangular prism.

Volume = length x width x height
Surface Area = (2 x length x width) + (2 x width x height) + (2 x length x height)

EXPLORE

Provide each group of 3 or 4 students with 24 cubes and an 8-inch (20-cm) diameter circle cut out of card stock. Make calculators, rulers, drawing paper, graph paper, and pencils available. Place the transparent sphere (optional) on display for students to access. (You may also conduct this activity using an overhead projector with an 8-inch [20-cm] diameter circle drawn on the transparency.) Students can take turns arranging the cubes on the transparency to find solutions. Ask students to use the blocks and card-stock circles and draw, or use any other method to figure out the possible dimensions of the cube.

Distribute the Student Handout sheets. Ask students to write all the possible dimensions of a rectangular prism that has a volume of 24 cubic inches (393 cubic cm).

Ask students to indicate in the last column of the table whether the rectangular prism will fit into the 8-inch (20-cm) diameter sphere.

If students are having difficulty visualizing a cube within a sphere, you may want to incorporate some visual aides. A transparent sphere that can open, such as a hamster ball, will allow you to place a cube inside to demonstrate the placement of the computer inside the PSA. Alternatively, you can use two equal sized colanders or bowls and hold them around a rectangular prism showing the "computer" inside the sphere.

EXPLAIN

Ask students: "Which of your rectangular prisms fit into the sphere?"

Ask students how they know that their rectangular prisms will or will not fit in the sphere. Ask students what properties of a sphere provide constraints or challenges to storing objects inside. Guide them to conclude that the sphere's round shape limits the amount of interior area that can be filled.

Ask students why they think NASA chose a spherical shape, even though it constrains the storage space. Guide them to conclude that the sphere's round shape also makes it safer, since it doesn't have any corners or edges.

Ask students what needs to be done to the rectangular prism in order for it to fit into the sphere.

Students may realize at this point that if they use decimals, they can find many more dimensions for the rectangular prism.

Allow students to round the value for the volume of the sphere, so that 23.99 and 24.02 are acceptable values.

EXPLORE

Ask students to calculate the surface area of their rectangular prisms and to write these values in the Student Handout sheets.

Ask students to try and come up with other numbers that will result in a rectangular prism with more surface area. Permit students to use calculators if they are available.

EXPLAIN

Tell students that we can use math statements to describe how one quantity changes when another changes. Ask students whether they see a pattern in how the numbers for the surface area of the prism change as its dimensions change.

Ask students: “What needs to happen to the length, width, and height in order to have more surface area?”

If students do not see the pattern (that a flat rectangular prism has more surface area than a thick one) ask them to rewrite their list in order of increasing surface area. Ask them what happens to the length and width as the surface area increases.

Ask students to write a letter to the NASA scientists working on the PSA. Students should give their recommendations for the dimensions of the PSA computer, and they should explain why they have chosen their designs. This explanation should include the relationship they observed between surface area and the dimensions of the prism. They should also include a description of the constraints they had to take into consideration and should describe both the benefits and challenges that a sphere’s special properties provide.

EXTEND

Tell students that NASA wants to store twenty spare PSA computers in the test environment of the International Space Station such that the surface area of the computers exposed to air is minimized. Ask students to decide if they will make a high, long, or wide stack of rectangular prisms to minimize the surface area exposed to air. Have students draw and explain their recommendation. In their explanation, students should explain why a flat rectangular prism has more surface area than a thick rectangular prism of equal volume.

Bonus Problem:
NASA scientists at the Space Food Systems Laboratory in Houston, Texas, are planning meals for astronauts onboard the International Space Station. In addition to planning astronaut meals, the scientists in this laboratory are responsible for designing packaging for the food they produce. Scientists are looking for a package that will give off the **least** amount of heat. The package must be a rectangular prism with a volume of 125 cubic centimeters.

EVALUATE

As a class, create an assessment rubric for the PSA computer dimension recommendation activity. Suggested criteria for the rubric include:

- Appropriate numbers for the length, width and height of the rectangular prism.
- Appropriate drawings and reasoning methods to determine whether the rectangular prism will fit into the 8-inch (20-cm) diameter sphere.
- Correct assessment as to whether the rectangular prism will fit into the sphere.
- Clear oral reasoning as to why a rectangular prism will or will not fit into the sphere.
- Appropriate calculations of surface area.
- Clear written presentation of results.
- Clear oral presentation of results.
- Clear explanation of constraints taken into consideration.
- Clear explanation of the benefits and challenges of a sphere’s special properties.
- Clear explanation of the relationship between surface area and the dimensions of a prism.

Use the rubric to assess students’ letters to NASA and ensure they have mastered the major concepts and math skills. Also assess students’ abilities to apply these concepts to the computer stacking problem.

Student Handout

Formulas
Surface Area = (2 x length x width) + (2 x width x height) + (2 x length x height)
Volume = length x width x height

Fitting a 24-cubic inch Rectangular Prism into an 8-inch sphere
Possible dimensions of a rectangular prism with a volume of 24 cubic inches (393 cubic cm):

Volume (cubic inches)	Length (inches)	Width (inches)	Height (inches)	Fit in an 8” diameter sphere? Yes/No
24				
24				
24				
24				
24				
24				
24				
24				

List the reasons why some rectangular prisms will fit into the sphere and others will not:

____
____
____
____
____
____
____

Surface area of the rectangular prisms that fit into an 8-inch (20-cm) diameter sphere:

Volume (cubic inches)	Length (inches)	Width (inches)	Height (inches)	Surface area of rectangular prism (square inches)
24				
24				
24				
24				
24				

24				
24				
24				
24				
24				

On a separate piece of paper, write a letter to NASA scientists working on the PSA. Give your recommendations for the dimensions of the PSA computer, and explain why you have chosen your design. Explain the relationship you observed between surface area and the dimensions of the prism. Also include a description of the constraints you had to take into consideration and describe both the benefits and challenges that a sphere's special properties provide.

Computer Stacking

NASA wants to store twenty spare PSA computers in the test environment of the International Space Station such that the surface area of the computers exposed to air is minimized. They want you to decide if they should make a high, long, or wide stack of computers. In the space below, tell the NASA scientists how they should stack the computers. Be sure to explain why you made this decision, and draw pictures.

Bonus Problem

NASA scientists at the Space Food Systems Laboratory in Houston, Texas, are planning meals for astronauts onboard the International Space Station. In addition to planning astronaut meals, the scientists in this laboratory are responsible for designing packaging for the food they produce. Scientists are looking for a package that will give off the **least** amount of heat. The package must be a rectangular prism with a volume of 125 cubic centimeters. In the table below, provide the dimensions for the best package to suit the scientists' needs. Show your work in the space provided.

Volume (cubic centimeters)	Length (centimeters)	Width (centimeters)	Height (centimeters)	Surface area of package (square centimeters)
125				

Reasons why the dimensions of your package are the best:

Answer Key

Formulas
Surface Area = (2 x length x width) + (2 x width x height) + (2 x length x height)
Volume = length x width x height

Fitting a 24-cubic inch Rectangular Prism into an 8-inch sphere
Possible dimensions of a rectangular prism with a volume of 24 cubic inches (393 cubic cm):

Volume (cubic inches)	Length (inches)	Width (inches)	Height (inches)	Surface Area (square inches)	Fits in an 8" diameter sphere? Yes/No
24	*24*	*1*	*1*	*98*	*No*
24	*12*	*2*	*1*	*76*	*No*
24	*8*	*3*	*1*	*70*	*No*
24	*6*	*4*	*1*	*68*	*Yes*
24	*6*	*2*	*2*	*56*	*Yes*
24	*4*	*3*	*2*	*52*	*Yes*
24	*5.6*	*2.06*	*2.08*	*54.9*	*Yes*
24	*5.5*	*2.1*	*2.08*	*54.7*	*Yes*

Computer Stacking
NASA wants to store twenty spare PSA computers in the test environment of the International Space Station such that the surface area of the computers exposed to air is minimized. They want you to decide if they should make a high, long, or wide stack of computers. In the space below, tell the NASA scientists how they should stack the computers. Be sure to explain why you made this decision, and draw pictures.

The NASA scientists should make a high stack of computers, because a high stack will conceal the prism faces with the most surface area.

Bonus Problem
NASA scientists at the Space Food Systems Laboratory in Houston, Texas, are planning meals for astronauts onboard the International Space Station. In addition to planning astronaut meals, the scientists in this laboratory are responsible for designing packaging for the food they produce. Scientists are looking for a package that will give off the **least** amount of heat. The package must be a rectangular prism with a volume of 125 cubic centimeters. In the table below, provide the dimensions for the best package to suit the scientists' needs. Show your work in the space provided.

Having the least surface area for the food package is important to minimize heat loss, because less of the food will be exposed to the outside temperature. Thus, it is important to have the lowest surface area to volume ratio possible. The best dimensions to minimize the surface area of any rectangular prism is to have length = width = height (i.e. a cube). For a package of volume 125 cubic centimeters, the minimum surface area occurs when length = width = height = 5 centimeters. The surface area for this package will be 150 square centimeters.

Sample Scoring Tool

4:
Calculations are correct and clearly presented.
Students correctly determine whether the dimensions for each computer will allow it to fit in the PSA.
Reasoning is logical and clear explanations are provided.
Oral and written presentations are clear.

3:
Most calculations are correct and attempts are made to present clearly.
Students almost always correctly determine whether the dimensions for each computer will allow it to fit in the PSA.
Attempts are made to reason logically and provide clear explanations.
Attempts are made to provide clear oral and written presentations.

2:
Some calculations are correct and attempts moderately clear.
Students sometimes correctly determine whether the dimensions for each computer will allow it to fit in the PSA.
Explanations demonstrate limited logical bases.
Oral and written presentation skills need improvement.

1:
Few calculations are correct and attempts are unclear.
Students rarely correctly determine whether the dimensions for each computer will allow it to fit in the PSA.
Explanations do not demonstrate understanding of lesson content.
Oral and written presentations do not effectively express results or reasoning.